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The First Quantization of Spin $\frac{3}{2}$ Field in de Sitter Space

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Abstract The de Sitter solution to the positive cosmological Einstein field equation has been viewed as a one-sheeted hyperboloid embedded in a five dimensional Minkowski space. To find Lagrangian equation of supersymmetry-group in the de Sitter space, the different spinor field's quantization have been demonstrated. In this work, the first quantization of spin $\frac{3}{2}$ field in the time-space de Sitter universe with ambient space notation has been done.

Keywords de Sitter space · Spin quantization · Supersymmetry algebra · Spinor field

1 Introduction

Some researchers ([1, 2] and references therein) have used the effective field theory which first developed during the 1960's for illustrating of the chiral symmetry breaking associated with pions, protons and neutrons. This field includes the degrees of freedom associated with the high energies in that unbroken symmetries are realized linearly, while spontaneously broken symmetries are realized nonlinearly. At first glance, an unbroken linearly and nonlinearly, respectively realized $N = 1$ and $N > 1$ supersymmetry. Such arguments can be occurred based on either the non-existence of Majorana spinors or independent charge-conjugate in de Sitter supergravity with even N [3, 4]. In continuous to what mentioned above,the transformation of bosons (fermions) supersymmetry to fermions (bosons) has been done in the de Sitter space with some researchers [1, 5]. These researchers have found the quantization of spinor fields in the de Sitter space by considering spin 0 and 1. Based on our knowledge, there is no report for spin $\frac{3}{2}$ quantization in the ambient space notion of de Sitter universe.

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2 Theory

In order to find spin $\frac{3}{2}$ field equations we first write Dirac equation in the curvature de Sitter space in an ambient coordination as below:

$$(-i \not{x}\gamma.\bar{\partial} + H^{-1}\lambda)\psi(x) = 0. \tag{1}$$

We start with (1) due to its simplicity form. In general, the group of de Sitter, G, contain two independent Casimir operators [1]. We have thus tried to determine the eigenstates of spin $\frac{3}{2}$ by introducing Hermitian generators related to Li-group algebra of $G = SO\,(1,4)$. By introducing the $L_{\alpha\beta}$ are the infinitesimal generators where $L_{\alpha\beta}$ obeys the following commutation relations:

$$\begin{aligned} &[L_{\alpha\beta}, L_{\gamma\sigma}] = -i(\eta_{\alpha\gamma}L_{\beta\sigma} + \eta_{\beta\sigma}L_{\alpha\gamma} - \eta_{\alpha\sigma}L_{\beta\gamma} - \eta_{\beta\sigma}L_{\alpha\gamma}), \\ &L_{\alpha\beta} = M_{\alpha\beta} + S_{\alpha\beta}. \end{aligned} \tag{2}$$

$M_{\alpha\beta}$ is orbital section:

$$M_{\alpha\beta} = -i(x_\alpha\partial_\beta - x_\beta\partial_\alpha) = -i(x_\alpha\bar{\partial}_\beta - x_\beta\bar{\partial}_\alpha). \tag{3}$$

$\bar{\partial}_\beta$ is tangential derivative on l'hyperboloid X_H and we have:

$$\bar{\partial}_\beta = \Theta_{\alpha\beta}\partial^\alpha = \partial_\beta + H^2 x_\beta x., \tag{4}$$

the $\Theta_{\alpha\beta} = \eta_{\alpha\beta} + H^2 x_\alpha x_\beta$ is projection "transverse". The tangential derivative and projection "transverse" have following properties:

$$\bar{\partial}_\beta x^2|_{x^2=-H^{-2}} = x^2\bar{\partial}_\beta|_{x^2=-H^{-2}}, \qquad \Theta_{\alpha\beta}x^\beta|_{x^2=-H^{-2}} = 0.$$

The form of the $S_{\alpha\beta}$ depends on the spin of the field. The spin $\frac{3}{2}$ combination of spin $\frac{1}{2}$ with eigenfunctions of spinor field and spin 1 with eigenfunctions of vector field. So the spin $\frac{3}{2}$ field must be simultaneous eigenfunction of operator $Q_1^{(1)}$ connected to the spin 1. We can thus define it as follow

$$S_{\alpha\beta}^{(\frac{3}{2})} = S_{\alpha\beta}^{(l)} + S_{\alpha\beta}^{(\frac{1}{2})}, \tag{5}$$

where

$$S_{\alpha\beta}^{(\frac{1}{2})} = -\frac{i}{4}[\gamma_\alpha, \gamma_\beta], \tag{6}$$

$$S_{\alpha\beta}^{(1)} = -i[\delta_{\alpha\gamma}k_\beta - \delta_{\beta\gamma}k_\alpha]. \tag{7}$$

Let us classify unitary irreducible representations of $SO\,(1,4)$ de Sitter group by using the eigenvalues of two casimir operators $Q^{(1)}$ and $Q^{(2)}$.

The possible values of join parameters, $\Delta(p = s = \frac{3}{2}, q = \frac{1}{2} + i\nu)$; $U^{s=\frac{3}{2},\nu}$ yields to

$$Q^{(1)} = \left[\left(\frac{9}{4} + \nu^2\right) - s(s+1)\right]I \quad \Longrightarrow \quad Q^{(1)}_{(\frac{3}{2})} = \left(\nu^2 - \frac{3}{2}\right)I, \tag{8}$$

$$Q^{(2)} = \left[\left(\frac{1}{4} + \nu^2\right) - s(s+1)\right]I \quad \Longrightarrow \quad Q^{(2)}_{(\frac{3}{2})} = \frac{15}{4}\left(\nu^2 + \frac{1}{4}\right)I. \tag{9}$$

We also consider casimir operator $Q^{(1)}$, because the equation of motion in physics is second order and field eigenvalues equation can be thus expressed as follow

$$Q^{(1)}_{\frac{3}{2}}\Psi_\lambda(x) = \left(\nu^2 - \frac{3}{2}\right)\Psi_\lambda(x). \tag{10}$$

From [1, 5, 6], we have

$$Q^{(1)}_{\frac{3}{2}} = -\frac{1}{2}L_{\alpha\beta}L^{\alpha\beta}, \tag{11}$$

where

$$Q^{(1)}_{\frac{3}{2}} = -\frac{1}{2}M_{\alpha\beta}M^{\alpha\beta} - \frac{1}{2}S^{(1)}_{\alpha\beta}S^{\alpha\beta(1)} - M_{\alpha\beta}S^{\alpha\beta(1)} - S^{(l)}_{\alpha\beta}S^{\alpha\beta(\frac{1}{2})}$$
$$- S^{(\frac{1}{2})}_{\alpha\beta}M^{\alpha\beta} - \frac{1}{2}S^{(\frac{1}{2})}_{\alpha\beta}S^{\alpha\beta(\frac{1}{2})}. \tag{12}$$

The first three terms are related to casimir operator of $Q^{(1)}_1$ and fourth term is given by [4, 7, 8]:

$$S^{(\frac{1}{2})}_{\alpha\beta}S^{\alpha\beta(l)}\psi_\lambda(x) = \psi_\lambda(x) - \gamma_\lambda(\gamma.\psi(x)). \tag{13}$$

We now need to calculate the two last terms of (11) by multiplying in eigenstates as below

$$Q^{(1)}_{\frac{3}{2}}\psi_\lambda(x) = \left(Q^{(1)}_l - l - \frac{5}{2} + \frac{i}{2}\gamma_\alpha\gamma_\beta M^{\alpha\beta}\right)\psi_\lambda(x) + \gamma_\lambda(\gamma.\psi_\lambda(x)), \tag{14}$$

in that from [3], we can write

$$Q^{(1)}_l\psi_\lambda(x) = Q^{(1)}\psi_\lambda(x) - 2\partial_\lambda x.\psi_\lambda(x) - 2\psi_\lambda(x), \tag{15}$$

$$Q^{(1)}_{\frac{3}{2}}\psi_\lambda(x) = \left(Q^{(1)}_0 - \frac{11}{2} + \frac{i}{2}\gamma_\alpha\gamma_\beta M^{\alpha\beta}\right)\psi_\lambda(x) - 2\partial_\lambda x.\psi_\lambda(x)$$
$$+ 2x_\lambda\partial.\psi_\lambda(x) + \gamma_\lambda(\gamma.\psi_\lambda(x)). \tag{16}$$

In order to find the solution of above equations, we use mass- unitary irreducible representations under conditions given as

$$\begin{gathered} x.\psi_\lambda(x) = 0, \\ \partial.\psi_\lambda(x). \end{gathered} \tag{17}$$

Taken into account that above conditions are valid in any time-space de Sitter space with ambient coordination [5]. Based on conditions of (15), we have

$$Q^{(1)}_{\frac{3}{2}}\psi_\lambda(x) = \left(-\frac{11}{2} + \frac{i}{2}\gamma_\alpha\gamma_\beta M^{\alpha\beta} - \frac{1}{2}M_{\alpha\beta}M^{\alpha\beta}\right)\psi_\lambda(x) + \gamma_\lambda(\gamma.\psi_\lambda(x)), \tag{18}$$

and using (10) and (18), the eigenvalues of $Q^{(1)}_{\frac{3}{2}}$ operator is

$$\left(\nu^2 - \frac{3}{2}\right)\Psi_\lambda(x) = \left(-\frac{11}{2} + \frac{i}{2}\gamma_\alpha\gamma_\beta M^{\alpha\beta} - \frac{1}{2}M_{\alpha\beta}M^{\alpha\beta}\right)\psi_\lambda(x) + \gamma_\lambda(\gamma.\psi_\lambda(x)). \tag{19}$$

From Dirac identity given by

$$-\frac{1}{2}M_{\alpha\beta}M^{\alpha\beta} = \left(\frac{1}{2}\gamma_\alpha\gamma_\beta M^{\alpha\beta}\right)^2 + \frac{3i}{2}\gamma_\alpha\gamma_\beta M^{\alpha\beta}, \tag{20}$$

and $\gamma.\psi_\lambda(x) = 0$, we can find the eigenvalues equation

$$\left(\left(\frac{1}{2}\gamma_\alpha\gamma_\beta M^{\alpha\beta} + 2i\right)^2 - \nu^2\right)\psi_\lambda(x) = 0. \tag{21}$$

Therefore, we obtain the Dirac-de Sitter equation as follow:

$$\left(\left(\frac{1}{2}\gamma_\alpha\gamma_\beta M^{\alpha\beta} + 2i\right) \pm \nu\right)\psi_\lambda(x) = 0, \tag{22}$$

where Dirac operator is

$$D = -\frac{i}{2}\gamma_\alpha\gamma_\beta M^{\alpha\beta} - 2. \tag{23}$$

In this case, the first orders of differential equations for particles with spin 3/2 are

$$(iD \pm \nu)\psi_\lambda(x) = 0. \tag{24}$$

The following relations are equivalent so that we can just get homogeneous answers of above equation for an arbitrary order of σ [1] as

$$Q_0^{(1)}\psi_\lambda(x) = -\sigma(\sigma + 3)\psi_\lambda(x). \tag{25}$$

Using eigenvalues of $Q_{\frac{3}{2}}^{(1)}$, we have

$$\begin{gathered} Q_{\frac{3}{2}}^{(1)}\psi_\lambda(x) = \left(\nu^2 - \frac{3}{2}\right)\Psi_\lambda(x), \\ \left(Q_0^{(1)} - 4 + \frac{i}{2}\gamma_\alpha\gamma_\beta M^{\alpha\beta} - \nu^2\right)\psi_\lambda(x) = 0, \\ -\sigma(\sigma + 3) - 2 - \nu^2 \pm i\nu = 0, \end{gathered} \tag{26}$$

where the answers of above equations are

$$\begin{aligned} \sigma &= -2 + i\nu, \\ \sigma &= -2 - i\nu. \end{aligned} \tag{27}$$

3 Conclusions

The formalism of the quantum field in de Sitter and Minkowski is similar. We could find the first quantization of spin $\frac{3}{2}$ field in the ambient coordination. This method can be used for determining the second quantization of spin $\frac{3}{2}$ field as well.

References

1. Garidi, T., Gazeau, J.P., Rouhani, S., Takook, M.V.: arXiv:gr-qc/0608004V1 (2006)
2. Bagger, J., Galpperin, A.: Phys. Rev. D **55**, 1091 (1997)
3. Bartesaghi, P., Gazeau, J.P., Moschella, U., Takook, M.V.: Class. Quantum. Gravity **18**, 4373 (2001)
4. Coleman, S., Mandula, J.: Phys. Rev. **159**, 1251 (1967)
5. Gazeau, J.P., Takook, M.V.: J. Math. **41**, 5920 (2000)
6. Lesimple, M.: Lett. Mat. Phys. **143**, 150 (1988)
7. Bicak, J., Krtous, P.: Phys. Rev. Lett. **88**, 211101 (2002)
8. Pahlavan, A., Rouhani, S., Takook, M.V.: arXiv:gr-qc/0506099v2, 9 Oct. 2005